Is bimaximal mixing compatible with the large angle MSW solution of the solar neutrino problem?

C. Giunti
INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I–10125 Torino, Italy
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It is shown that the large angle MSW solution of the solar neutrino problem with a bimaximal neutrino mixing matrix implies an energy-independent suppression of the solar ν_e flux. The present solar neutrino data exclude this solution of the solar neutrino problem at 99.6% C.L. [S0556-2821(99)02007-X]

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The possibility that the neutrino mixing matrix U has the bimaximal mixing form

$$U=\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{1}$$

has attracted a great deal of attention [1] after the presentation of the Super-Kamiokande evidence in favor of atmospheric neutrino oscillations with large mixing [2].

Neutrino bimaximal mixing is capable of explaining in a elegant way the atmospheric neutrino anomaly [2–5], through $\nu_\mu\to\nu_\tau$ oscillations due to[1] $\Delta m^2_{31}\sim10^{-3}$ eV^2 and the solar neutrino problem (SNP) [6–10] through $\nu_e\to\nu_\mu,\nu_\tau$ oscillations in vacuum due to $\Delta m^2_{21}\sim10^{-10}$ eV^2 [11,12].

As noted in [13], the results of the recent analysis of solar neutrino data presented in [14] seem to imply[2] that neutrino bimaximal mixing may be also compatible at 99% C.L. with the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) [15] solution of the SNP [16,23] (see Fig. 2 of [14]).

Here I would like to notice that this conclusion seems to be in contradiction with the exclusion at 99.8% C.L. of an energy-independent suppression of the solar ν_e flux presented in the same paper [14] (see Sec. IV D).

The reason of this incompatibility is that bimaximal mixing with the $\Delta m^2_{21}\sim10^{-5}-10^{-4}$ eV^2 corresponding to the LMA solution of the SNP implies an energy-independent suppression by a factor 1/2 of the solar ν_e flux.

This can be seen following the simple reasoning presented in [17]. The mixing of the neutrino states in vacuum is given by (see, for example, [18])

$$|\nu_\alpha\rangle=\sum_{k=1,2,3}U^*_{\alpha k}|\nu_k\rangle \quad (\alpha=e,\mu,\tau), \tag{2}$$

where the states $|\nu_\alpha\rangle$ $(\alpha=e,\mu,\tau)$ describe neutrinos produced in weak interaction processes and the states $|\nu_k\rangle$ $(k=1,2,3)$ describe neutrinos with definite masses m_k.

In the bimaximal mixing scenario the numbering of the massive neutrinos is the usual one, i.e., such that $m_1\leqslant m_2\leqslant m_3$, and $\Delta m^2_{31}\sim10^{-3}$ eV^2 for the solution of the atmospheric neutrino anomaly. If $\Delta m^2_{21}\sim10^{-5}-10^{-4}$ eV^2 for the LMA solution of the SNP, we have $\Delta m^2_{32}\simeq\Delta m^2_{31}\sim10^{-3}$ eV^2.

Solar neutrinos have energy $E\sim1$ MeV and the ratio $\Delta m^2_{31}/E\simeq\Delta m^2_{32}/E\sim10^{-9}$ eV is much larger than the matter induced potential $V\lesssim10^{-11}$ eV in the interior of the sun. Hence, the evolution equation of the heaviest massive neutrino ν_3 is decoupled from that of the two light neutrinos ν_1 and ν_2 (see, for example, [19]). Taking also in account that in the case of bimaximal mixing $U_{e3}=0$, one can see that an electron neutrino is created in the core of the sun as a superposition of the two light mass eigenstates ν_1 and ν_2 and, whatever happens during his propagation in the interior of the sun, its state when it emerges from the surface of the sun is a linear combination of $|\nu_1\rangle$ and $|\nu_2\rangle$:

$$|\nu\rangle_S=\sum_{k=1,2}a_k|\nu_k\rangle, \tag{3}$$

with

[1]$\Delta m^2_{kj}\equiv m^2_k-m^2_j$ is the difference between the squared masses of the two massive neutrinos ν_k and ν_j. In the bimaximal mixing scenario there are three massive neutrinos, ν_1, ν_2 and ν_3.

[2]I want to emphasize from the beginning that I do not want to criticize at all the beautiful paper [14]. I am only concerned with the interpretation of its results.

$$|a_1|^2+|a_2|^2=1. \tag{4}$$

Since the massive neutrino states $|\nu_k\rangle$ propagate as plane waves, the state describing the neutrino detected on the Earth is

$$|\nu\rangle_E=\sum_{k=1,2} a_k e^{-iE_kL}|\nu_k\rangle, \tag{5}$$

where L is the distance from the surface of the Sun to the detector on the Earth. The survival probability of solar electron neutrinos is then given by $P_{\nu_e\to\nu_e}=|\langle\nu_e|\nu\rangle_E|^2$:

$$P_{\nu_e\to\nu_e}=\left|\sum_{k=1,2} a_k e^{-iE_kL}\langle\nu_e|\nu_k\rangle\right|^2=\left|\sum_{k=1,2} a_k e^{-iE_kL}U_{ek}\right|^2. \tag{6}$$

Taking now into account the explicit values $U_{e1}=1/\sqrt{2}$ and $U_{e2}=-1/\sqrt{2}$ in the case of bimaximal mixing and the fact that the neutrinos are extremely relativistic, we have

$$P_{\nu_e\to\nu_e}=\frac{1}{2}\left|a_1-a_2\exp\left(-i\frac{\Delta m^2_{21}L}{2E}\right)\right|^2. \tag{7}$$

In the case of the LMA solution of the SNP $\Delta m^2_{21}\sim10^{-5}-10^{-4}$ eV^2 and the survival probability (7) oscillates with an oscillation length $4\pi E/\Delta m^2_{21}\sim10^7$ cm that is about one million times smaller than the Sun-Earth distance. Hence, the oscillations are not observable on the Earth because of averaging over the energy spectrum and only the average probability

$$\langle P_{\nu_e\to\nu_e}\rangle=\frac{1}{2}(|a_1|^2+|a_2|^2)=\frac{1}{2} \tag{8}$$

is observable. We have obtained the announced result: *the LMA solution of the SNP in the bimaximal mixing scenario implies an energy-independent suppression of the solar* ν_e *flux of a factor* 1/2.

Therefore, we have the apparent paradox that an energy-independent suppression of the solar ν_e flux seems to be allowed at 99% C.L. by Fig. 2 of Ref. [14] and is excluded at 99.8% C.L. in Sec. IV D of the same paper. Notice that the two conclusions are based on the same set of data and the same theoretical calculation of the neutrino flux produced by thermonuclear reactions in the core of the sun [20].

The fact that the two cases refer to the same physical situation, i.e., an energy-independent suppression of the solar ν_e flux, is also shown by the χ^2 calculated in the two cases. The χ^2 of the right border of the LMA region[3] in Fig. 2 of Ref. [14] is $4.3+9.2=13.5$, whereas the χ^2 calculated in Sec. IV D of the same paper for an energy-independent suppression of the solar ν_e flux by a factor 0.48 is 12.0. Since this is the best fit for an energy-independent suppression of the solar ν_e flux, a value of $\chi^2=13.5$ for a suppression factor 0.5 looks plausible.

The solution of the apparent paradox explained above lies in *a correct statistical interpretation of the allowed LMA region in Fig. 2 of Ref. [14] and of the exclusion in Sec. IV D of the same paper*. The two cases have different statistical meanings.

The allowed regions in Fig. 2 of Ref. [14] are obtained under the assumption that the neutrino masses and mixing parameters are not known. In this case a general neutrino oscillation formula is used in the fit, with the neutrino masses and mixing angles considered as free parameters. The best fit in the LMA region happens to have a $\chi^2_{\rm min}=4.3$, which corresponds to a C.L. of 3.8% with 1 DOF. Hence, a LMA solution is allowed at 3.8% C.L. and one can draw a 99% C.L. region corresponding to the parameters that have $\chi^2\leqslant\chi^2_{\rm min}+9.2$.

The statistical analysis discussed in Sec. IV D of Ref. [14] assumes that the solar ν_e flux is suppressed by a constant factor that is the free parameter to be determined by the fit. It happens that the best fit has $\chi^2_{\rm min}=12.0$, which corresponds to a C.L. of 0.2% with 2 DOF. Hence, the hypothesis is excluded at 99.8% C.L. and no allowed region of the free parameter can be drawn.

Since the two statistical analyses start from different assumptions, it is clear that they answer different questions and their conclusions cannot be compared. Moreover, it is important to notice that the test of the bimaximal mixing scenario with $\Delta m^2_{21}\sim10^{-5}-10^{-4}$ eV^2 does not correspond to either of the two statistical analyses. Indeed, if this scenario is assumed, we *know* that the solar ν_e flux is suppressed by an energy-independent factor 0.5 and there is no parameter to fit. Hence we test the hypothesis under consideration on the basis of its χ^2. The $\chi^2\simeq13.5$ indicated by Fig. 2 of Ref. [14] implies a C.L. of 0.4% with 3 DOF. Therefore, the hypothesis is rejected at 99.6% C.L.

Notice that this exclusion is based only on the values of the elements U_{e1}, U_{e2} and U_{e3} of the neutrino mixing matrix. This means that also other types of neutrino mixing matrix, as those discussed in [21], are incompatible with the LMA solution of the SNP.

In conclusion, I would like to emphasize that the allowed regions of the neutrino oscillation parameters calculated in the usual way (i.e., as Fig. 2 of Ref. [14]) cannot be used to test a definite model (as the bimaximal mixing model) because they have been obtained under different assumptions.[4] The appropriate tool for testing such a model is the goodness-of-fit test [22].

I would like to thank Z.Z. Xing for bringing my attention to the problem under discussion.

[3]If $U_{e3}=0$, we have $\sin^2 2\vartheta=4|U_{e1}|^2|U_{e2}|^2$ (see [12]) and $\sin^2 2\vartheta=1$ corresponds to $|U_{e1}|=|U_{e2}|=1/\sqrt{2}$, as in the bimaximal mixing matrix (1).

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